Special Relativity in High-Energy Particles

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Outline

Topics to be covered:

- Introduction to Special Relativity
- Background on High-Energy Particle Physics
- Theoretical Framework:
  - Time Dilation
  - Length Contraction
- Lorentz Transformation
- Momentum and Mass
- Types of Particle Accelerators
- Corrections in Particle Accelerators
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Introduction to Special Relativity

Special relativity is a phenomenon involving the motion of bodies through space-time. This ‘space-time’ refers to a concept involving not only the movement of objects within a certain space, but also time periods of their own. Time here is not an absolute concept in which it would remain consistent for all inertial frames (static or dynamic observers), rather opting to shift according to these inertial frames. Therefore, special relativity is an explanation for shifts in mass, space, and time as effects of speed.

While discussing special relativity, four crucial points must be acknowledged. First, the speed of light ‘c’ remains constant for all inertial frames, as proved by the Michelson-Morley experiment [1]. Second, the dynamic observer must not know they are moving in the sense that all other elements behave in respect to them (Galileo’s relativity) [2]. Third, there is no such absolute motion. The idea of one object moving depends entirely on the inertial frame which will be further discussed in time dilation. Lastly, an ‘event’ refers to an occurrence happening in one point in space-time. The effects of special relativity may be calculated only for these ‘events’.

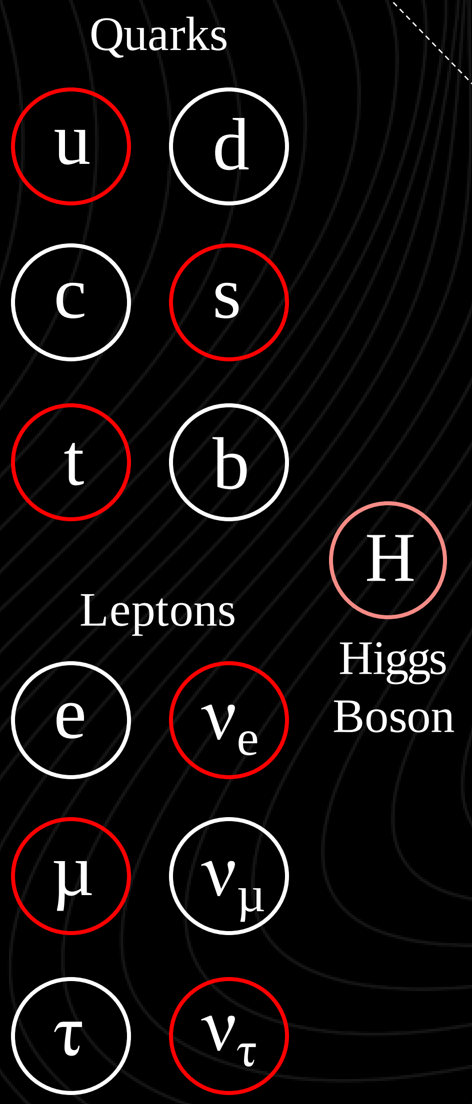
In this presentation, we will largely focus on special relativity’s effects on space and time with respect to high-energy particles. For simplicity, the example we will be using throughout is a static (unmoving) observer Y and a dynamic (moving) observer Z.

Background on High-Energy Particle Physics

Particle physics is a branch of physics studying the nature of the fundamental particles. It deals with the observation of the interactions between particles as well as the detection of new particles. According to the Standard Model, there are two groups into which particles are classified: quarks (up, down, strange, charm, top, bottom) and leptons (electron, electron neutrino, muon, muon neutrino, tau, tau neutrino) [3]. Aside from these, there is also the Higgs boson.

This branch also studies the four fundamental forces and their relations with particles. These forces are strong force, weak force, gravitational force and electromagnetic force. However, gravitational force is excluded while dealing with subatomic particles [3].

The behaviour of these particles is studied at high speeds. They are accelerated by particle accelerators such as the Large Hadron Collider (LHC) among others. Since these particles traverse near the speed of light ‘c’ they reach relativistic speeds. This results in significant changes in particle behaviour due to special relativity. In this presentation, we will be discussing the differences in particle lifetimes as well as trajectories and how particle accelerators are calibrated to accommodate these.



Quarks

u d

c s

t b

Leptons

e ν_e

μ ν_μ

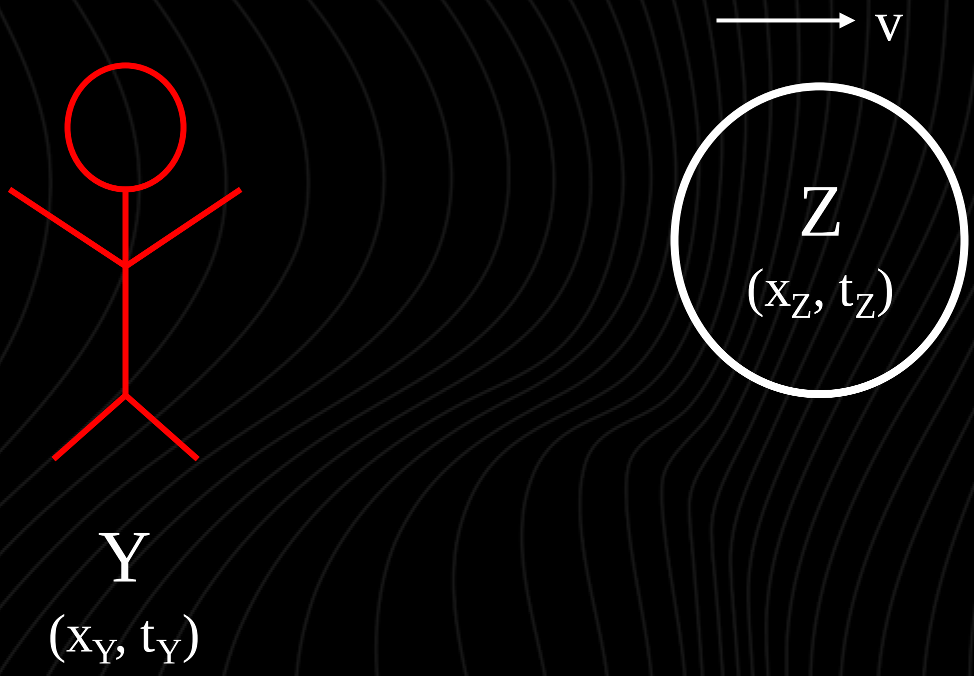
τ ν_τ

H

Higgs Boson

Theoretical Example

Throughout this presentation, we will be using the example of a static observer Y and a moving particle Z. Z has a velocity ‘v’ and the spatial and time coordinates of Y and Z are (x_Y, t_Y) and (x_Z, t_Z), respectively.



Time Dilation

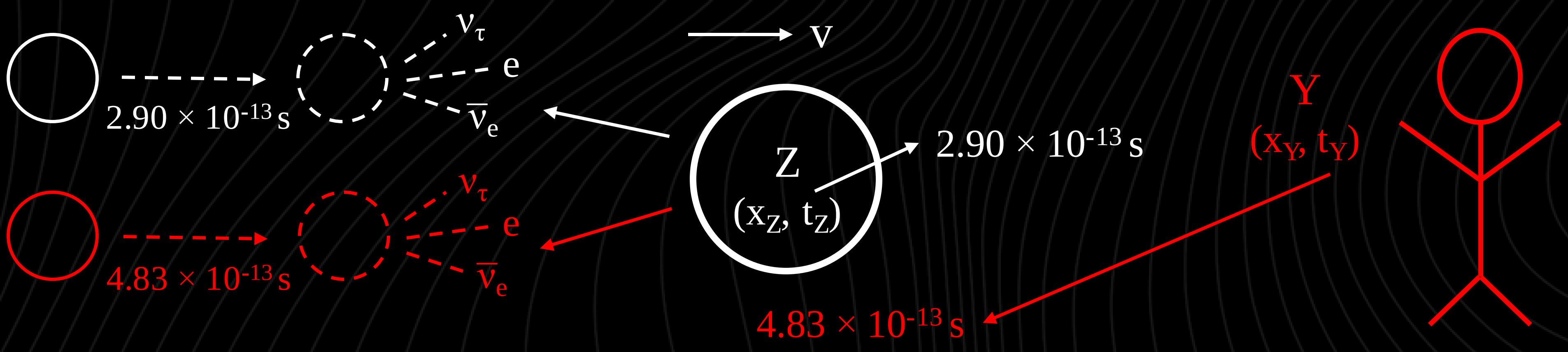
Time dilation is the resultant effect on time and one of the fundamental effects of special relativity. As previously discussed, time has no absolute value and changes with inertial frames. This suggests that the time experienced by the two observers will be different with respect to their individual velocities. Furthermore, the 'time coordinates' of Y and Z, which without relativity's effects should be the same, are found to be different. This is how the idea of non-absolute time arises, where time itself is relative to the observer. The inertial frames in this example are that of Y, where particle Z is moving, and that of Z, where the entire environment around it is moving and it is at rest.

After a few simple calculations involving different inertial frames, one finds that $\Delta t_Y = \gamma . \Delta t_Z$ where Δt_Z is the time experienced by particle Z and Δt_Y is the time change of Z as observed by person Y. $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, a formula for γ used in special relativity, where 'v' is the velocity of the moving body and $\gamma \geq 1$. This means that the time duration observed by the static observer (Δt_Y) is always greater than the time duration experienced by the dynamic observer (Δt_Z). Hence, the term 'time dilation' comes into view.

Time Dilation

In practical experimental setups, time dilation enforces a direct change in the lifetime of particles being observed. This poses a risk in the misinterpretation of interactions and the laboratory clock. The same calculation ($\Delta t_Y = \gamma . \Delta t_Z$) is applied here for particle lifetimes. Consider particle Z to be a tau (τ), a type of lepton. The lifetime of a tau is approximately 2.9×10^{-13} seconds [4]. This implies that the tau takes 2.9×10^{-13} seconds to decay into an electron, tau neutrino, and electron antineutrino [5].

While particle Z moves at velocity ‘v’ its lifetime in its own inertial frame (Δt_Z) will remain to be 2.9×10^{-13} seconds as the laws of physics are the same in all inertial frames. However, in the inertial frame of observer Y, the lifetime of the particle ‘dilates’. This is due to special relativity where time is relative to speed. Considering that $v = 0.8 \times c$, the lifetime of the tau with respect to observer Y can be obtained. This value Δt_Y is approximately 4.83×10^{-13} seconds, which is greater than the tau lifetime experienced by the particle itself.



Length Contraction

Similar to time, space is affected by special relativity. However, where time for the moving body slows down, the distance covered shortens with respect to a static observer. This conclusion has been reached through a theoretical light ruler travelling in a moving vehicle where light is emitted, reflected and detected [6]. Since after reflection the light travels in the direction opposite to the motion of the vehicle, the distance covered appears to reduce for an outside observer [6]. Hence the term ‘length contraction’.

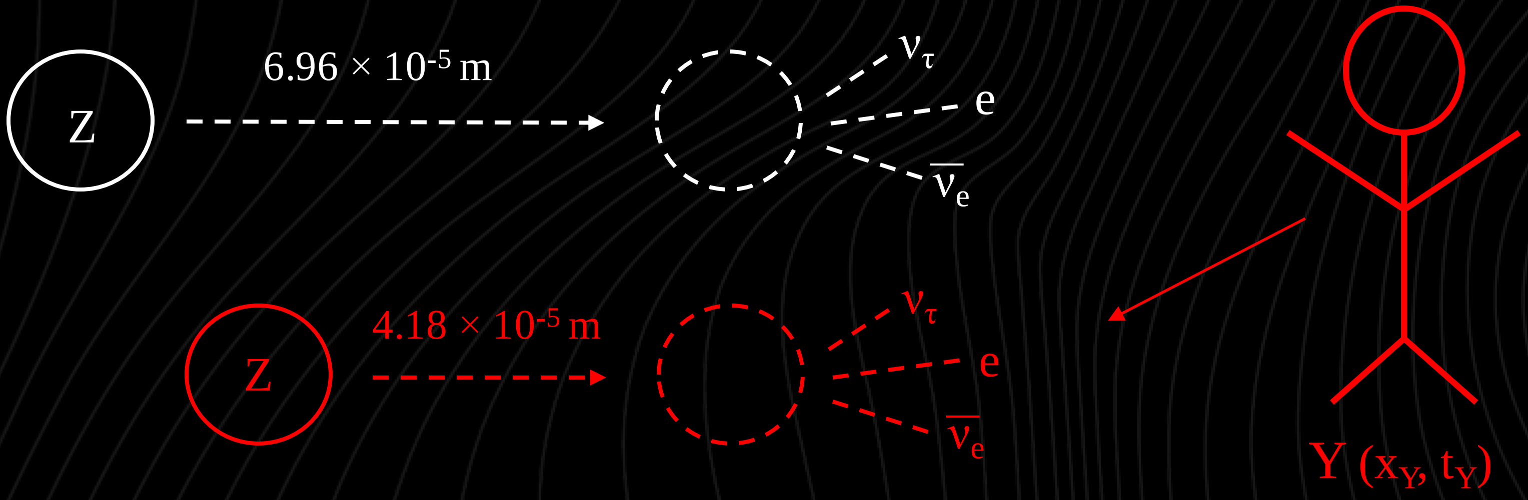
This reduction in length is represented by the equation $D_Y = \frac{D_Z}{\gamma}$ where the distance travelled with respect to particle Z is D_Z and that with respect to observer Y is D_Y. Again, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ where $\gamma \geq 1$.

Because the value of gamma is greater than one, the distance observed by person Y is less than the distance traversed by particle Z by a factor of gamma.

Length Contraction

In the example of Y and Z where Z (tau) is moving a certain distance at velocity ‘v’ before decaying, the question is how far the tau will travel before its lifetime is over. Consider that the tau is moving in a linear path for simplicity. On account of the laws of physics maintaining the same in all inertial frames [2], D_Z is the same as a regular calculation of distance. $D_Z = v \times \Delta t_Z = 6.96 \times 10^{-5}$ meters.

On the other hand, the outside observer Y perceives this distance to contract or become shorter by a factor of gamma. $D_Y = \frac{D_Z}{\gamma} \approx 4.18 \times 10^{-5}$ meters. This value is less than that experienced by the tau itself, hence proving length contraction.



Comparative Graph

While the time period perceived by the observer Y is greater than that of particle Z, the distance proves to be the opposite.

This concludes that time dilation increases the observed lifetime of particles while length contraction reduces the perceived trajectory of these particles in the inertial frame of a static observer (Y).

Both increase/decrease by a factor of γ. [Note: The graph has no value on a comparative basis other than to form a contrast between decrease and increase as the units are different (meters and seconds).]

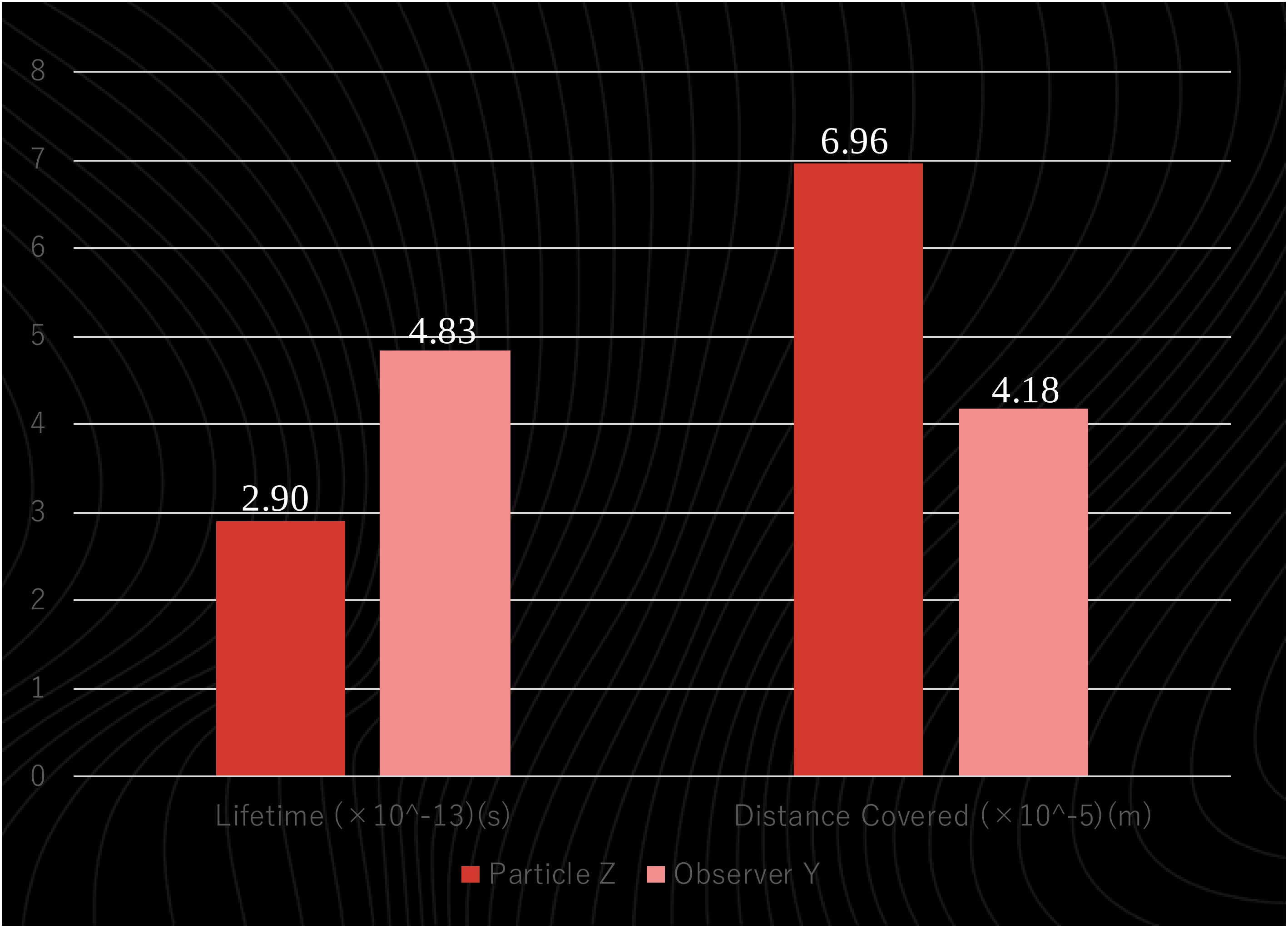


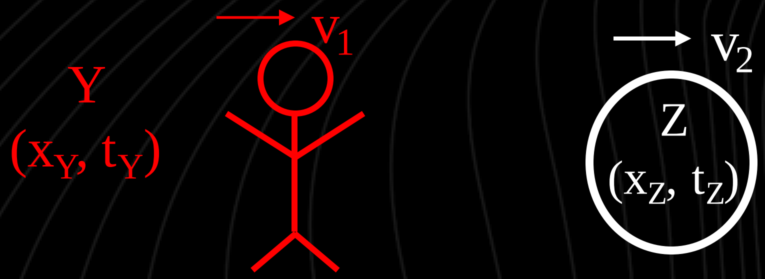
fig: Comparative Graph between Time Dilation and Length Contraction

Lorentz Transformation

In the previous understanding of calculating time dilation and length contraction, the simple case of Y and Z was taken. This is where Y is a static observer and Z is a dynamic body. The effects of time dilation and length contraction are visible to observer Y by a factor of γ ($\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$).

However, if both Y and Z are moving at individual constant speeds v_1 and v_2, the observed time and distance is bound to change. This cannot be derived simply using the factor of gamma because those equations account for the observer having a relative speed ‘v’ equal to zero.

This is where the Lorentz Transformation comes into play. It is a set of equations to derive the time dilation and length contraction in the inertial frame in the case where both observers are moving at a constant speed [7]. The position of each body is taken in four coordinates x, y, z, t, and in the other observer’s frame, x’, y’, z’, t’ [8]. Four equations are used to calculate the coordinates in the observer’s frame [8].



Momentum and Mass

According to the law of conservation of momentum, the total momentum must remain the same before and after collision of bodies. The formula for momentum is $p = mv$ for an object with ‘m’ mass and ‘v’ velocity. However, it is found in time dilation that the time observed increases by a factor of gamma, hence changing the value of the object’s velocity. This in turn violates the law of conservation of momentum.

Einstein later concluded that the mass of an object must increase as its velocity nears the speed of light ‘c’ to keep momentum constant and fulfil the law. This describes an object moving at constant velocity ‘v’ to have inertia corresponding to its ‘new’ mass [9].

The equation for the relativistic mass ($m_{relative}$) corresponding to a rest mass ‘m’ is the following:

$$m_{relative} = m\gamma \text{ where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

As the speed of an object nears the speed of light ‘c’, the absolute limit, its relativistic mass becomes a multitude of times greater than its rest mass or initial mass [9]. This is observed in particle accelerators in which particles move at speeds near ‘c’.

Particle Accelerators → Cyclotrons

Particle accelerators are devices used to propel subatomic (or atomic) particles to high speeds to observe their effects on collision with a nucleus, isotopes, etc. For the purposes of special relativity, the types to be discussed are cyclotrons and synchrotrons.

- Cyclotron:

  This type of particle accelerator consists of two semi-circular electrodes (dees) between the two poles of a magnet. The dees have a gap between them, which is the region where particles are accelerated. The subatomic particles are generated at the centre of the accelerator and an electric field forces them into one of the dees. The magnetic field constantly allows the particles to move in a circular motion at their constant speed in the dee.

  The electric field changes the polarization of the gap between dees so that once the particles reach this gap, they are accelerated into the next dee, increasing their orbit's radius as well as their speed at each crossing. Until the effects of special relativity become more significant, the mass and strength of the magnetic field remain constant, concluding that the frequency of gap crossings is also constant.

[10]

Particle Accelerators → Synchrotrons

- Synchrotron:

  This type of particle accelerator maintains the radius of the particles by strengthening the magnetic field. This uses a much smaller magnet than a cyclotron and radio-frequency voltages cause the acceleration. A linear particle accelerator feeds this the particles, which can be accelerated at a steady rate. The rate of increase of strength of the magnetic field effects the rate of energy increase in particles.

  To focus the beam of particles, the magnetic field must be shaped in a proper way. Using these principles, synchrotrons can propel high-energy beams.

  These particle accelerators also overcome the barriers imposed on speed by special relativity, which will be explored in the next slide.

[11]

Corrections in Particle Accelerators

Since particles are accelerated near the speed of light ‘c’, this is seen to divert from Newtonian mechanics, where motion and time are absolute. As results of special relativity, time dilation and length contraction have large effects on the interpretation of high-energy particle experiments. To avoid error in calculating results from these experiments, particle accelerators must be calibrated to accommodate special relativity.

In account of time dilation and length contraction, the Lorentz Transformation and factor of gamma must be used for two dynamic bodies and one dynamic body, respectively.

In cyclotrons, the limit for energy of protons by acceleration is 25 million electron volts [10]. This is by cause of the mass increase of the particles due to special relativity. Because of this mass increase, the orbital frequency decreases such that the electric field changes to decelerate rather than accelerate the particles [10].

However, to adjust for relativistic mass increase, a device called a synchrocyclotron must be utilized. Using this synchrocyclotron, the frequency of alternating electric fields can be changed in accordance with the accelerated particles, thereby ensuring that the polarity changes as the particles approach the gap. Second, a device known as an azimuthally-varying-field cyclotron may be used. This adjusts the magnetic field near the edges of the dees to strengthen the field and focus it by azimuthal variation [10].

Conclusion

Special relativity, a concept relying entirely on speed, has proved many early concepts wrong, such as the idea of mass and time being absolute. It is now a crucial key to discovering new aspects of physics and solving the previously inexplicable behaviour of high-energy particles.

Furthermore, the application of special relativity in particle physics is tremendous; these particles move at near-light speed, undergoing significantly different time periods, trajectories, and masses. While the observation period of the lifetime of particles like taus increases by a severe factor of gamma, this results in disastrous effects in the interpretation of these particles from their own inertial frame, where the entire environment is in motion and they are at rest.

For a particle in motion, its ‘rest frame’ time period, length, as well as mass can be calculated using the Lorentz Transformation or the factor of gamma. In particle accelerators such as synchrotrons, the strength of magnetic fields or frequency of alternating electric fields can be adjusted to accommodate the mass gain of particles at relativistic speeds.

Special relativity itself is the first concept to unify space and time into one space-time continuum, allowing for the theories formulated in present times. If not for its consideration of speed’s effects, major experiments carried out, especially in particle accelerators, would be at severe risk of extreme misinterpretation.

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